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AN ELECTRONIC INDICATOR FOR ANGULAR VELOCITY AND ACCELERATION

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SUMMARY

An electronic circuit for instantaneously indicating angular velocity and acceleration, both positive and negative, of rotating masses is described; readings are taken on electrical indicating meters. The circuit may be used in applications where sudden changes in acceleration are not encountered. The instrument has the advantage of putting almost no drag on the rotating member.

The indicator is described in terms of general application. A circuit diagram, calibration curves, and a mathematical analysis of the differentiating circuit are included. An example showing use of the indicator with the four-ball wear top is included to illustrate a specific application. In this application, the error in the reproducibility of the indications was less than 5 percent.

INTRODUCTION

A program involving a great number of lubrication tests with a modified Shell Development Company four-ball wear top suggested the need for a simpler and quicker method of obtaining data from the machine than that provided by the manufacturer. In the four-ball wear top the lubricant is tested by observing the angular velocity and deceleration of a loading weight slowed down by frictional forces between four balls lubricated by the oil under test. References 1 and 2 give descriptions of tests with the four-ball wear top. (See fig. 1 for sectional view of the machine.) In the manufacturer's method the angular velocity and acceleration were determined by counting the cycles of a direct inking oscillograph and by plotting a curve of angular velocity as a function of time. From this curve the slopes were measured to give the angular acceleration.

An electronic indicator was developed at the Aircraft Engine Research Laboratory of the NACA from February to September 1943 that greatly reduces the time and effort required to obtain these

data. It can be used to show the average frequency of a series of nearly evenly spaced electrical impulses, the mean frequency of which does not exceed 150 cycles per second. The circuit is so designed that it is not affected by moderate changes in the magnitude and shape of the pulses. If the frequency approximates a predescribed function of time, the indicator will also show the magnitude of the time derivative of frequency.

Because the indicator is not restricted in its use, it is herein described in terms of general application. An example showing operation of the indicator with the four-ball wear top is also given to illustrate its adaptability. Calibration curves and a mathematical analysis of the differentiating circuit are included.

DESCRIPTION AND APPLICATION OF THE INDICATOR

Description and Operation

The electronic indicator for angular velocity and acceleration, a circuit diagram of which is shown in figure 2, derives its power from the 117-volt a-c. line and uses gas-filled and vacuum tubes of a common variety. In operation a series of positive voltage pulses, proportional to the speed of the rotating member under observation, is applied to the grid of the thyratron V_2 across the resistor R_1. Each positive pulse trips the thyratrons V_2 and V_3. The average current flowing through V_3 is measured by the velocity meter M_1. The voltage developed across R_8 by this current is directly proportional to the number of thyratron trips per unit time and is therefore a measure of the angular velocity of the rotating member. After the voltage has been filtered by R_9, C_6, R_{10}, and C_7, it is applied to the differentiating circuit C_8 and R_{12}. The voltage appearing across R_{12}, within the limits to be discussed in the section Mathematical Analysis, is the time derivative of the voltage across R_8. The differentiated voltage is amplified by V_4 and measured as a change in the plate current of the amplifying tube by the acceleration meter M_2. A reading less than zero on M_2 indicates that the rotating member is decelerating. In figure 2 the meter M_2 is connected to give positive readings of deceleration.

The voltage developed across R_8 must be a linear function of the angular velocity in order that the same meter reading on M_2 be obtained for the same acceleration independent of the

magnitude of the velocity, as will be shown in the following section. If the voltage across the plate-load resistor of a thyratron is to be a linear function of the number of impulses applied to the grid, three conditions must be fulfilled:

1. The number of grid impulses and the number of plate-current pulses must be in a one-to-one correspondence.

2. The thyratron must be self-quenching.

3. All plate-current pulses must be of equal magnitude regardless of the nature or frequency of the grid impulses.

In order that these three conditions be fulfilled over the range of speeds that the indicator was called upon to measure (0 to 150 pulses per sec), two thyratrons were necessary. The first thyratron V_2 is self-quenching for nearly any shaped pulse and transmits a positive voltage of almost constant magnitude to the second thyratron V_3 even though the initiating pulses may vary somewhat in size and shape. The second thyratron V_3 provides a plate current linear with the frequency of the impulses applied to its grid.

The two voltage-regulator tubes V_5 and V_6 provide constant voltage for the second thyratron V_3 and the amplifying tube V_4. These tubes aid materially in rendering the velocity scale linear and in making the acceleration reading independent of the velocity.

Application

From the preceding discussion it can be seen that the source of the positive pulses proportional to the velocity of the rotating member and applied across R_1 is immaterial. The pulses may be obtained, for example, from a battery in series with a contactor attached to the rotating member, from a magneto, or from an alternating-current generator driven by the rotating member.

Another means of generating positive pulses can be obtained by subjecting a photoelectric cell to light pulses from a mirror rotating with the member under consideration. (See fig. 3.) This method, which is the one used in the application described herein, has the distinctive advantage of placing almost no additional drag on the member in order to secure the desired voltage pulses. Another interesting application of this same method is used with the Dornier Air Log described in reference 3.

MATHEMATICAL ANALYSIS

In the circuit used for the instrument the mechanical inertia of the velocity meter is the only factor that limits the indication of rapid changes in angular velocity. However, the differentiating circuit was designed to give the time rate of change of a voltage varying in a manner described by the following equation:

$$e = kv = k\left(v_o + at + bt^2\right) \tag{1}$$

where

e	instantaneous voltage across R_6
v	instantaneous velocity
v_o	initial velocity
t	time
a, b, k	constants

The voltage e can be differentiated by a series resistor-condenser combination (see reference 4) as shown by the following analysis:

Let

R	series resistor across which the differentiated voltage will appear
C	series condenser
i	current through R and C

Then

$$e = Ri + \frac{1}{C}\int i\,dt = k\left(v_o + at + bt^2\right) \tag{2}$$

When $t = 0$, the velocity is constant and the voltage on the condenser is equal to kv_o. The solution of equation (2) is

$$i = kC\left[a + 2b\,(t - RC) - (a - 2RCb)\,\epsilon^{-t/RC}\right] \tag{3}$$

and

$$\frac{di}{dt} = kC\left[2b + \left(\frac{a}{RC} - 2b\right)\epsilon^{-t/RC}\right] \tag{4}$$

The voltage iR and hence the current i should be proportional to the time derivative of the applied voltage. Any departure from this proportionality must be considered an error in the differentiating circuit. This error may be computed as follows:

From equation (2)

$$\frac{i}{C} = \frac{de}{dt} - R\frac{di}{dt} \tag{5}$$

Dividing by de/dt

$$\frac{\frac{i}{C}}{\frac{de}{dt}} = 1 - \frac{R\frac{di}{dt}}{\frac{de}{dt}} \tag{6}$$

From equation (6) it is clear that the expression $R\dfrac{di/dt}{de/dt}$ represents the fractional error in assuming that i is proportional to de/dt. But from equation (5)

$$\frac{R\frac{di}{dt}}{\frac{de}{dt}} = \frac{1}{1 + \dfrac{i}{RC\frac{di}{dt}}} \tag{7}$$

Substituting the expressions of equations (3) and (4) into the right-hand member of equation (7)

$$\text{Percentage error} = \frac{2bRC + (a - 2bRC)\,\epsilon^{-t/RC}}{a + 2bt} \times 100 \tag{8}$$

In the circuit used the time constant RC was 1 second. In order that the percentage error be small, b must be small in comparison with a, and t must be about five times RC. If the velocity is in the form given in equation (1), if the acceleration changes slowly with time, and if acceleration readings are not taken until after the acceleration has been in progress for at least 5 seconds, accurate acceleration readings can be obtained.

By a similar analysis it can be shown that the circuit is suitable for indicating acceleration if the velocity can be represented by the following equation:

$$v = \Sigma a_n \cos\omega_n t \tag{9}$$

provided $RC\omega_n$ is small in comparison with 1. In this particular circuit RC is equal to 1 and ω_n must therefore be small in comparison with 1.

In order that the measurement of the derivative of the voltage developed across R_8 be indicative of the deceleration, a linear relation must exist between the velocity and the voltage. To illustrate:

If

$$v = f(e)$$

then

$$\text{Acceleration} = \frac{dv}{dt} = \left(\frac{df}{de}\right)\left(\frac{de}{dt}\right) \tag{10}$$

In order that de/dt, the quantity measured, be proportional to dv/dt, df/de must be a constant. A linear relation between the voltage and the velocity must exist for df/de to be a constant.

OPERATION WITH FOUR-BALL WEAR TOP

The operation of the indicator with the modified four-ball wear top is relatively simple. The double-pole, single-throw switch S_1 (shown in fig. 2 as S_{1_a} and S_{1_b}) is open and the three-pole, two-position switch S_2 (shown in fig. 2 as S_{2_a}, S_{2_b}, and S_{2_c}) is closed when the unit is plugged into the 117-volt a-c. line. The light system is then so adjusted that the photocell is flooded with light when the mirror is in a favorable position. After the optical adjustment has been made, the light source is temporarily shut off. Switch S_1 is then closed. After the indicator has had several minutes to warm up, switch S_2 is opened and the meter M_2 is adjusted to zero by means of the variable resistor R_{14}. Switch S_2 is then closed, the light source is turned on, and the rotating member is brought up to speed.

The sensitivity control R_4 is advanced from an extreme negative position only sufficiently to give a steady reading on the velocity meter M_1. The driving torque is next removed from the rotating member and switch S_2 is opened. (Switch S_2 had been left closed so that the condenser C_8 might charge more quickly through R_{11} than through the sum of R_9, R_{10}, and R_{12}.) After the short initial time period, when the error is high, the angular velocity and deceleration of the rotating member are given by meters M_1 and M_2, respectively.

During the series of lubrication tests that followed the development of this instrument, its reproducibility was frequently checked. The velocity indications could be reproduced to the limit of the meter scale, that is, about 0.4 percent of full scale. The acceleration meter was not so reproducible. The readings varied as much as 5 percent between different runs at the same deceleration.

Use of the indicator greatly reduced the time required to obtain data with the four-ball wear top. By means of the electronic indicator, data for a single run could be prepared and plotted in 20 minutes; whereas 8 hours were required previously.

CALIBRATION OF METERS

The instrument can be calibrated by connecting a variable-frequency oscillator to R_1 through a small coupling condenser. At this time R_6, mounted inside the chassis, should be so adjusted for proper bias on thyratron V_3 that it trips each time the thyratron V_2 is fired by the oscillator. The variable resistor R_6 need not be readjusted until new tubes are installed. The velocity meter is calibrated when a plot of meter reading as a function of frequency has been made, as shown in figure 4. This relationship must be a straight line.

The acceleration meter M_2 may be calibrated by making a typical test run. As the rotating member changes its speed, readings of the two meters are taken at definite time intervals. By means of figure 4 the readings of M_1 are converted into angular-velocity values and a curve of velocity as a function of time is plotted. Figure 5 shows three curves taken with the four-ball wear top with three different loading weights. The negative accelerations were graphically determined for each time an acceleration-meter reading was recorded. These M_2 readings were then plotted against the graphically computed accelerations. A calibration of the acceleration meter is shown in figure 6.

CONCLUDING REMARKS

The electronic indicator obtains angular velocity and acceleration data without adding appreciable drag to the revolving member. The time required for preparing and plotting the data for a single run is about 20 minutes. In a number of test runs with the four-ball wear top the instrument readings were reproducible to 0.4 percent of full scale on the velocity indicator and to ±5 percent on the acceleration meter.

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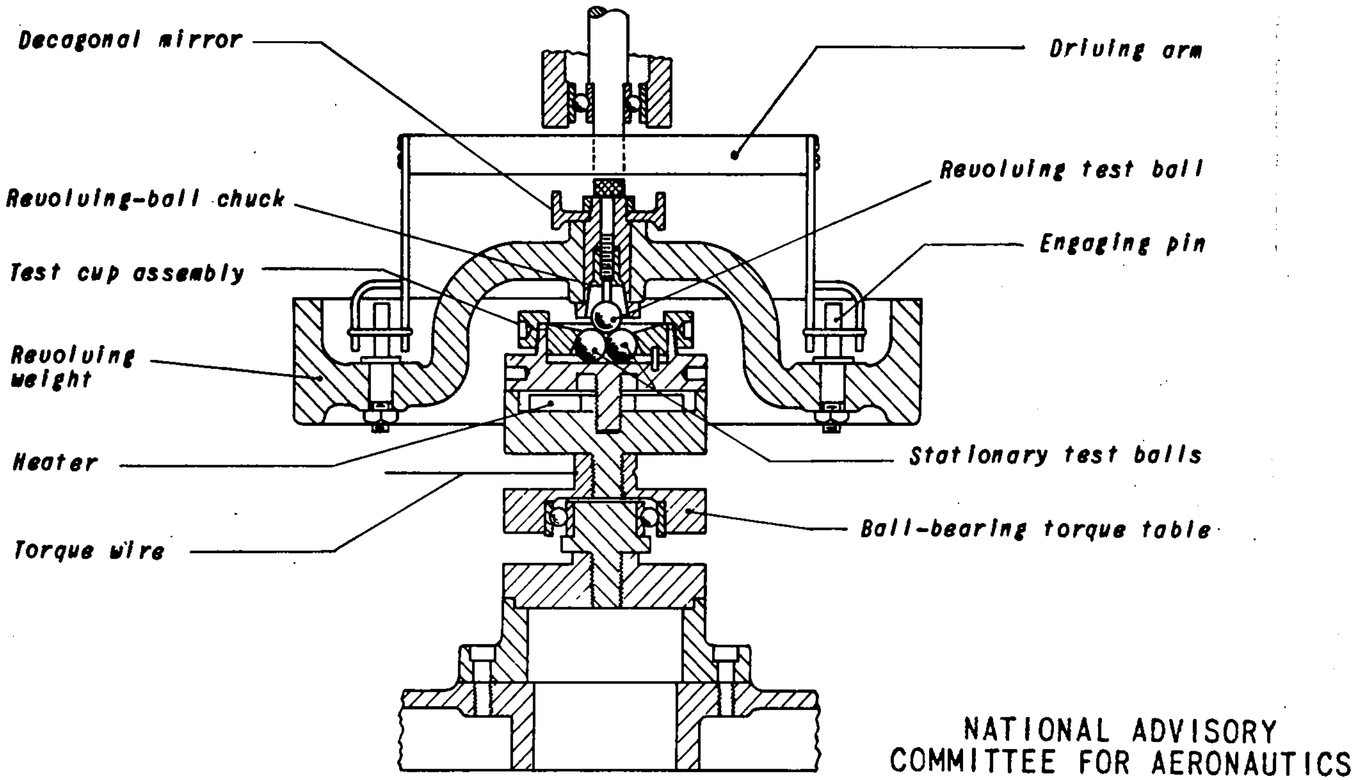

Figure 1. - Sectional view of modified four-ball wear top.

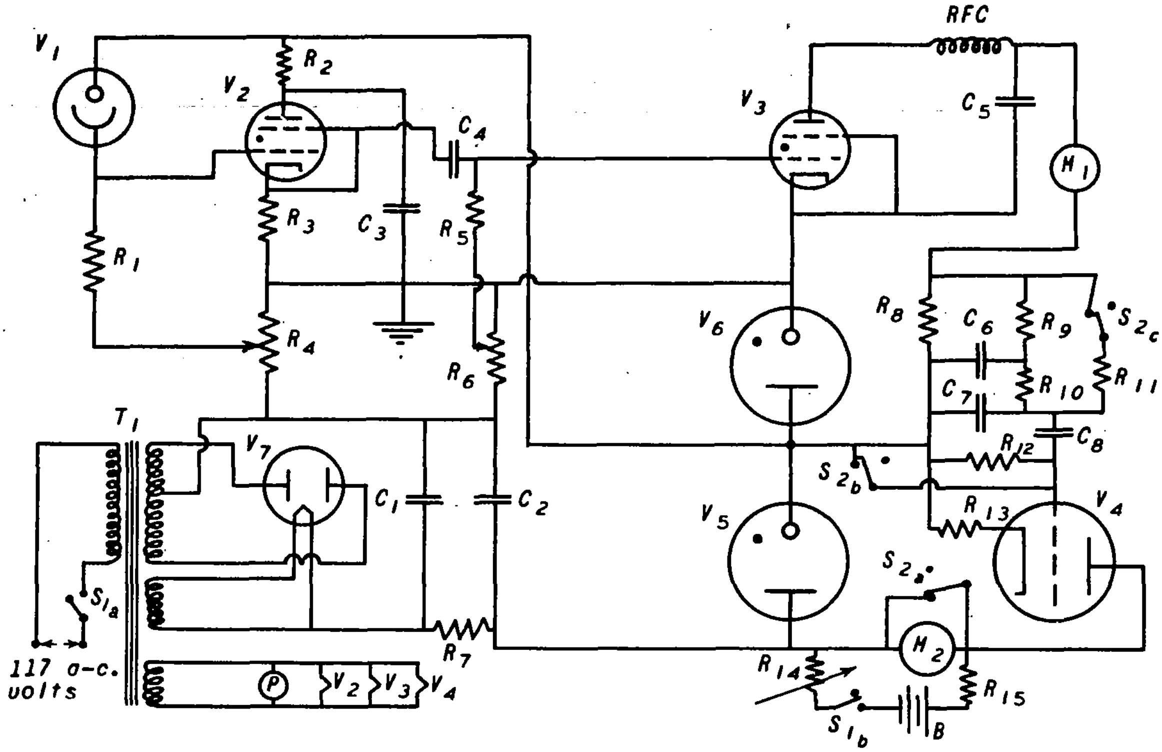

Figure 2. - Circuit diagram of the electronic indicator for angular velocity and acceleration. All resistors except R_7 are 2 watts; R_7 is 10 watts.

V_1	919
V_2, V_3	2051
V_4	6J5-GT
V_5	VR105-30
V_6	VR150-30
V_7	5U4-G
R_1	5,000,000 ohms
R_2, R_5	250,000 ohms
R_3	15 ohms
$^aR_4$, $^aR_6$	2,000 ohms
R_7	2,000 ohms
R_8	3,000 ohms
R_9, R_{10}	50,000 ohms
R_{11}	65,000 ohms
R_{12}	200,000 ohms
R_{13}, R_{15}	100 ohms
$^aR_{14}$	100 ohms

C_1, C_2	4.	microfarads
C_3	.005	microfarads
C_4	.0005	microfarads
C_5	.5	microfarads
C_6, C_7	8.	microfarads
C_8	5.	microfarads

T_1	Power transformer
RFC	7-millihenry choke
S_{1_a}, S_{1_b}	Double-pole, single-throw switch
S_{2_a}, S_{2_b}, S_{2_c}	3-pole, 2-position switch
B	1.5-volt battery
P	6.3-volt pilot light
M_1	0 to 15 milliamperes
M_2	0 to 500 microamperes

^{a}Variable resistors.

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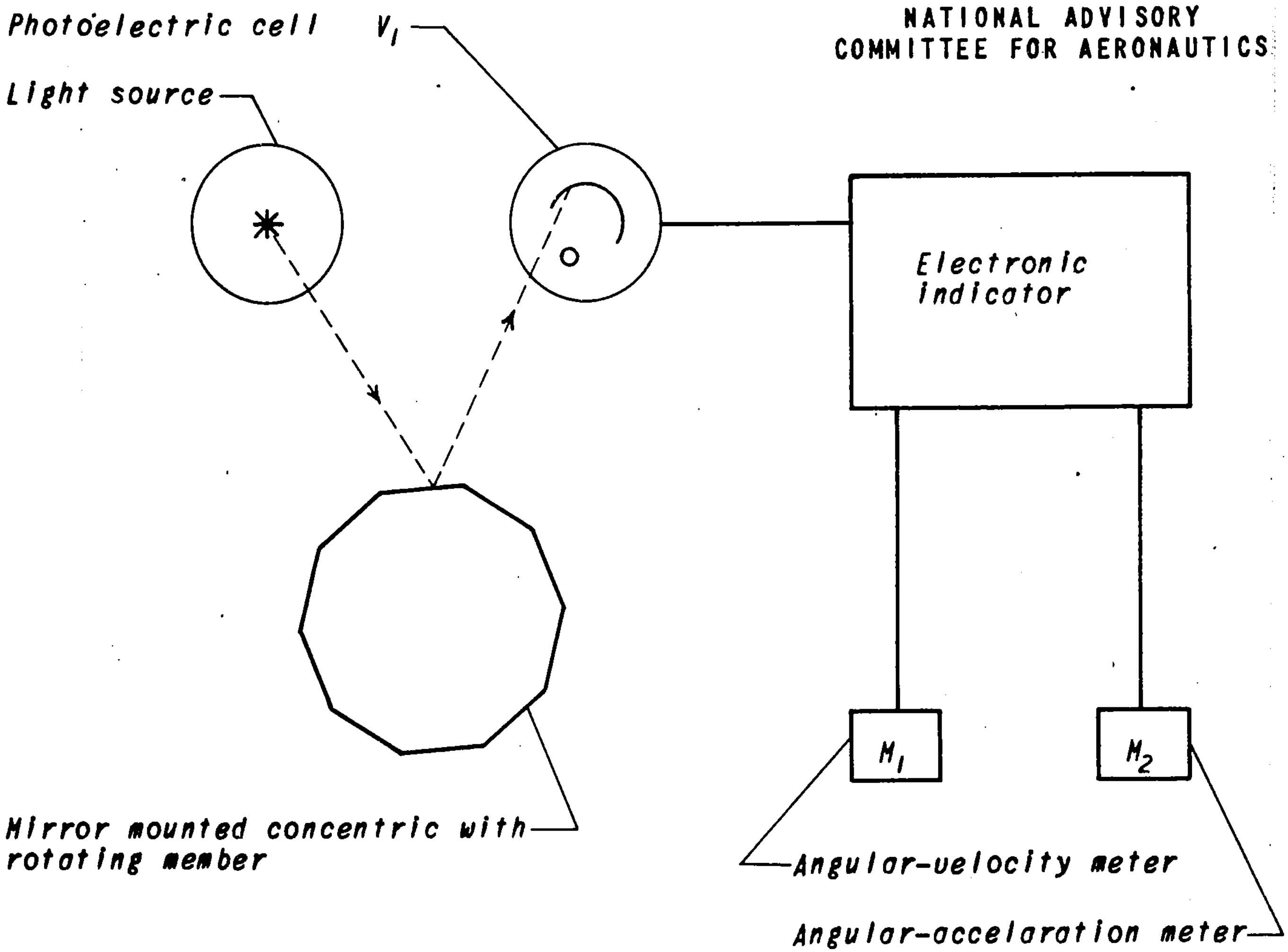

Figure 3. - Block diagram showing no-drag system for generating voltage pulses for electronic indicator.

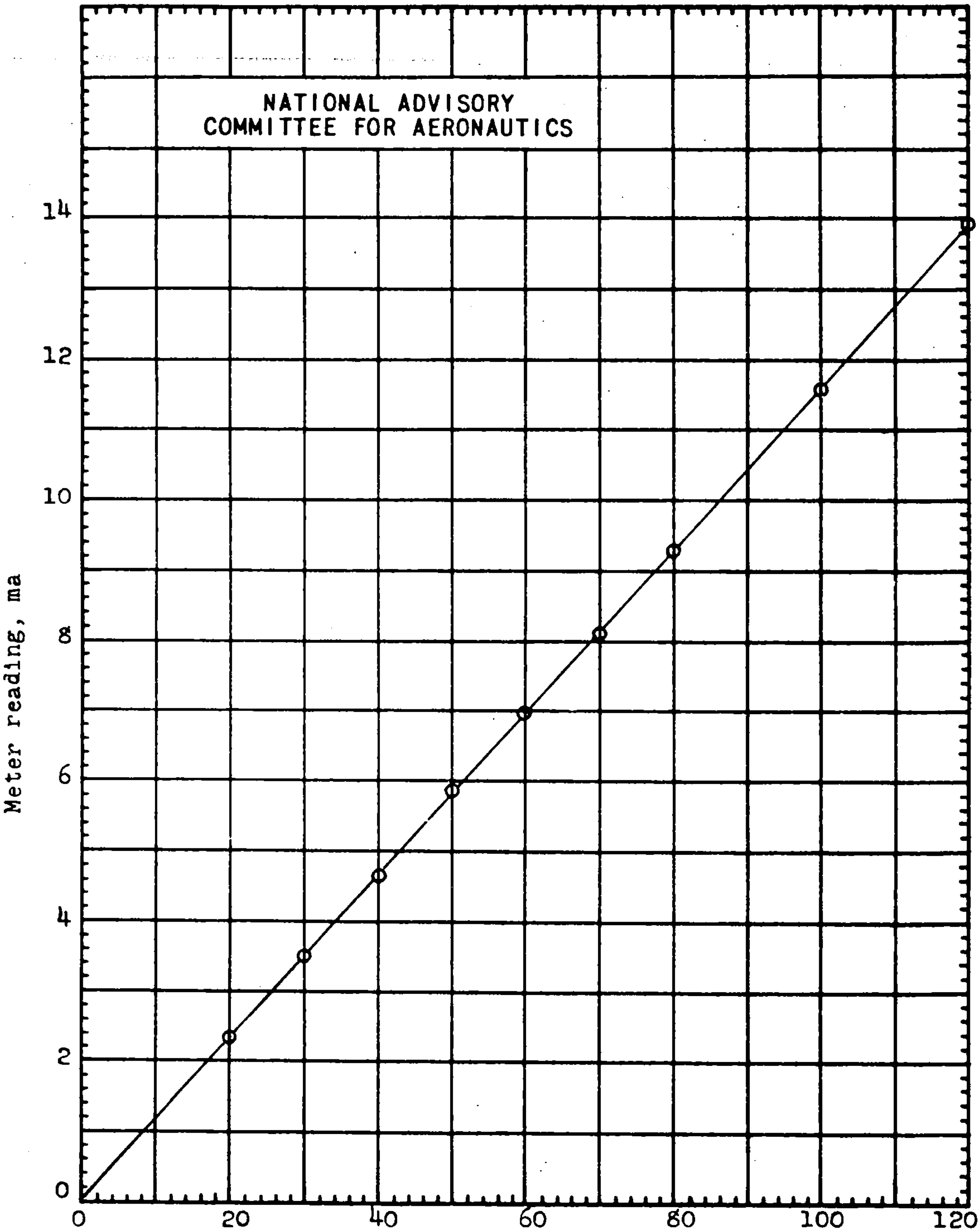

Figure 4. - Calibration of meter M_1.

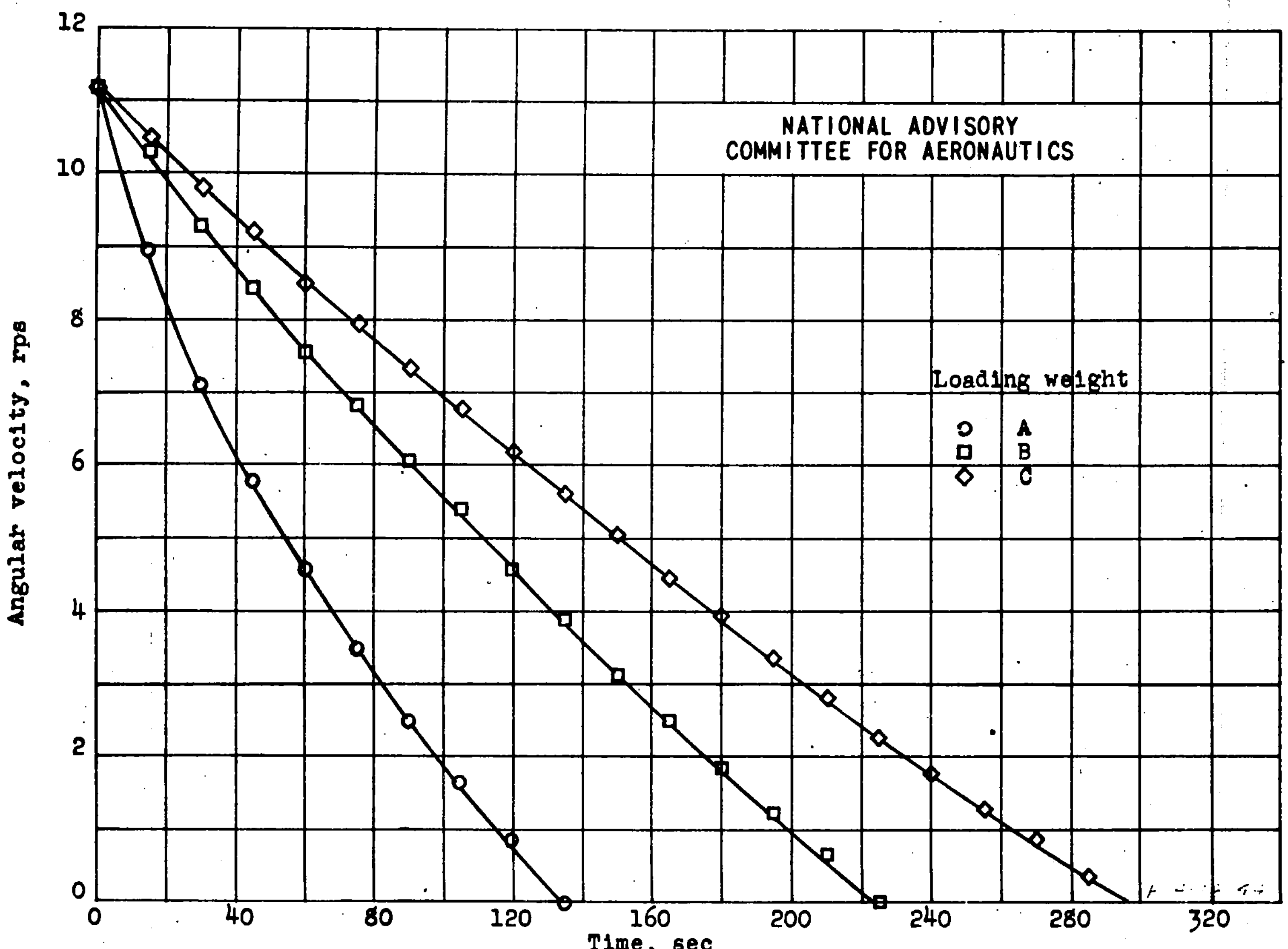

Figure 5. - Typical angular velocity-time curves of tests with three different loading weights of the four-ball wear top.

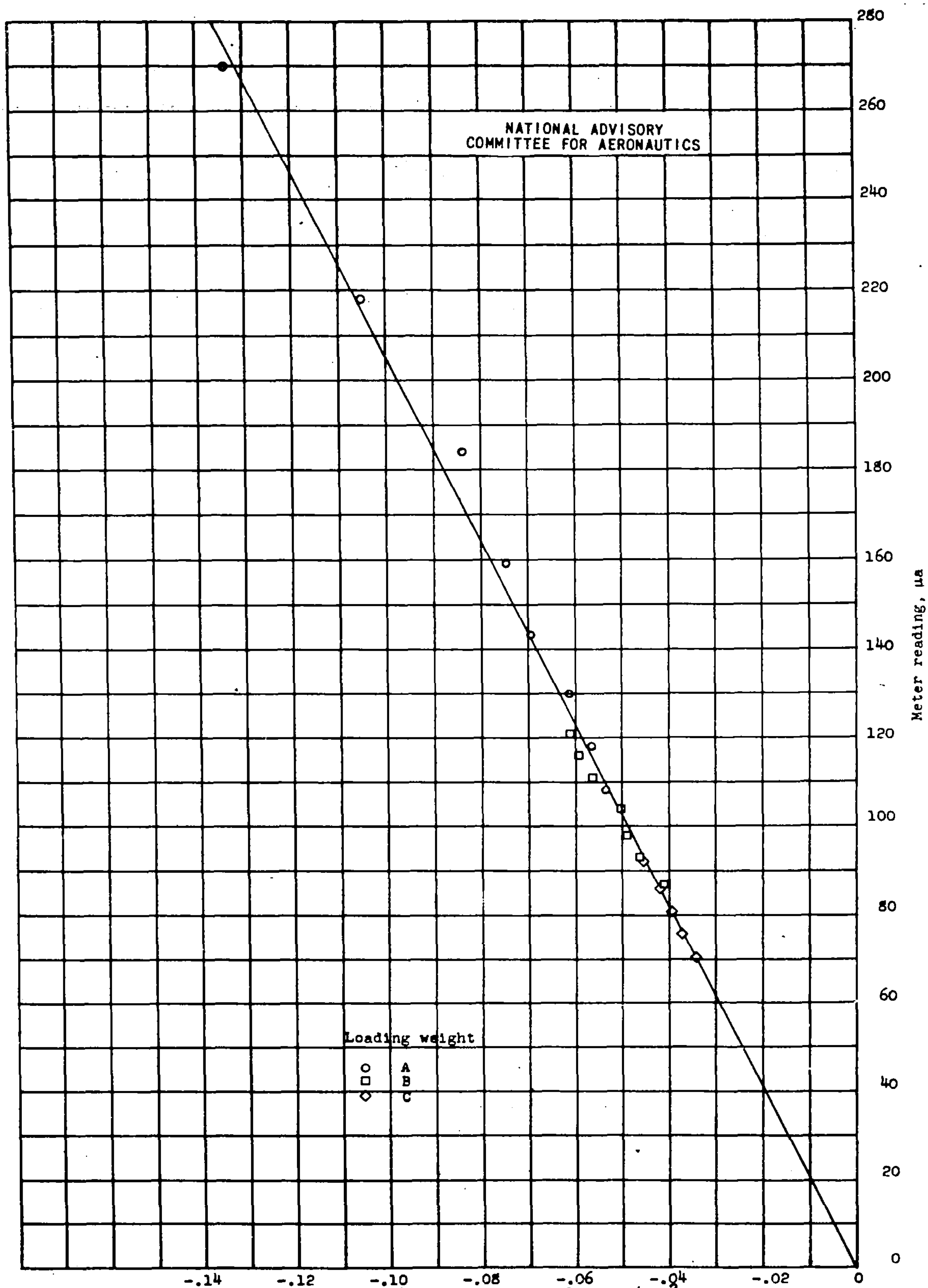

Figure 6. - Calibration of meter M_2.